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Mathematics News Letter

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Dedicated to mathematics in general and to the following aims in particular: (1) a study of the common problems of secondary and collegiate mathematics teaching, (2) a true valuation of the disciplines of mathematics, (3) the publication of high class expository papers on mathematics, (4) the development of greater public interest in mathematics by the publication of authoritative papers treating its cultural, humanistic and historical phases.

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CONTENTS

The Human Side of Augustus DeMorgan

*Within our circle of books read and re-read is none of rarer charm or more delightful humanism, than the two now classic volumes, *"A Budget of Paradoxes." It was written by the English Mathematician Augustus DeMorgan, who lived from 1806 to 1871. More courageous or independent mind than DeMorgan's does not adorn English history. Though educated at Cambridge, says the historian Cajori, "his scruples about the doctrines of the established church, prevented him from proceeding to the M. A. degree." Though he wrote a treatise on Formal Logic and a calculus which is still standard, his fame rests largely upon a vigorous crusade which he waged in class-room and in published article, for a more complete coordination of logic and mathematics. In season and out of season he preached that they should be made as inseparable as the two eyes of one's head.*

It is likely that a close study would discover that this logic instinct was responsible for DeMorgan's numerous excursions into fields void of technical mathematics but rich in matter that invited to the exercise of sharpened faculties of analysis. His appetite for the paradox-ranging from Michael Scott's Devils all the way to squaring the circle—was but an outcropping of this faculty. In close union with it was a deep and ever lively humor. Both his analysis and his humor found vent in the endless incongruities of human nature and human opinion. While his logic was busy setting them straight his sense of amusement over them never failed. Note the exquisite combination of philosophy and humor in the following paragraph taken from his "On Some Philosophical Atheists": "The absolute denial of a ruling power was not in the plan of the higher philosophers. It was left for the smaller fry. A round assertion of the non-existence of anything which stands in their way is the refuge of a certain class of minds; but it succeeds only with things subjective; the objective offers resistance. A philosopher of the appropriative class tried it upon the constable who appropriated him. 'I deny your existence' said he. 'Come along all the same' said the unpsychological policeman."

Again, from the same chapter is the following story told in DeMorgan's own racy language. It concerns Diderot, the philosopher, and Euler, the mathematician: "Diderot paid a visit to the Russian Court at the invitation of the Empress. He conversed very freely and gave the younger members of the Court circle a good deal of lively

*Second edition, edited by David Eugene Smith, and published by the Open Court Publishing Co. 1915.

atheism . . . So the following plot was contrived. Diderot was informed that a learned mathematician was in possession of an algebraic demonstration of the existence of God and would give it to him before all the Court if he desired to hear it. Diderot gladly consented . . . Euler advanced towards Diderot and said gravely and in a tone of perfect conviction: "Sir, $\dfrac{(a+b^n)}{n} = x$, *whence God exists; answer." Diderot to whom algebra was Hebrew, was embarrassed and disconcerted; while peals of laughter rose on all sides. He asked permission to return to France at once, which was granted."*

More of wisdom than of humor is in the following item of the DeMorgan "Budget": "There is much truth in the assertion that new knowledge hooks on easily to a little of the old, thoroughly mastered. The day is coming when it will be found that crammed erudition, got up for examinations, does not cast out any hooks for more."

Iconoclastic in his contempt of conventions as such. DeMorgan boasted (p. 251, Vol. 2) a pride in being "one of the class of rational paradoxers," a class he defined as "all who in private life and in matters which concern themselves take their own course no matter what people may think of them." One day he "bought a comb made of lead and intended to dye the hair." Let him tell what he did with it. "I divided the comb into two, separated the part of closed prongs from the other and thus I had two ruling machines. The lead marks paper and by drawing the end of one of the machines along, I could rule twenty lines at a time quite fit to write on. I thought I could have killed a friend to whom I explained it—he could not for the life of him understand how leaden lines on paper would dye the hair."

Not all the items of the DeMorgan "Budget" have a mathematical setting. The philosopher in him roamed over too wide an area for this. However, as he was primarily a mathematician, it was natural that most of them should at least represent a mathematical nucleus. Thus we find him turning the fiercest blasts of his ridicule upon the class of irrational paradoxers that is made up of circle squarers, angle trisectors and inventors of perpetual motion. Ever bellicose in the cause of truth, unlike the great majority of present day mathematicians who ignore the tribe, he flattered their egocentric ignorance far beyond any possible deserving by challenging their pretentions. Unluckily, the same impenetrable casing of conceit that prevents the species from seeing the

truth about quadrature or angle tri-section, also seems to keep them from recognizing when their theories have been demolished. We cite from the "Budget" (pp. 104-105) just one instance of his satire against the tribe. The offender was one "James Smith, Arch Paradoxer." Says DeMorgan: "He is beyond a doubt the ablest head at unreasoning and the greatest hand at writing it of all who have tried in their day to attach their name to an error. Common cyclometers sink into puny orthodoxy by his side.

The behavior of this singular character induces me to pay him the compliment which Achilles paid Hector, to drag him around the walls again and again. He had been treated with unusual notice and in the most gentle manner. The un-named mathematician E. M. bestowed a volume of mild correspondence upon him; Rowan Hamilton quietly proved him wrong in a way accessible to an ordinary school boy. Whewell, as we shall see, gave him the means of seeing himself wrong even more easily than by Hamilton's method. Nothing would do; it was small kick and silly fling at all; and he exposed his conceit by alleging that he, James Smith, had placed Whewell in the stocks. He will therefore be universally pronounced a proper object of the severest literary punishment; but the opinion of all who can put two propositions together will be that of the many strokes I have given Smith the hardest and most telling are my re-publication of his own attempts to reason."

—S. T. S.

Louisiana State University Mathematics Club Organized

The graduate students of the Department of Mathematics of Louisiana State University met with the faculty of the department December 7, 1934, for the purpose of organizing a Mathematics Club, which in the future will be sponsored by the graduate mathematics students of the University.

The purpose of the Club is to stimulate a greater interest in mathematics; to encourage worthy mathematical research; and to afford opportunity for mutual fellowship among its members.

THE GRADUATE MATHEMATICS CLUB,
RUTH JOHNSON, *President*.

Louisiana-Mississippi Section and Council Programs

The joint meeting of the Louisiana-Mississippi sections of the National Council of Teachers of Mathematics and of the Mathematical Association of America will be held in Jackson, Miss., on Friday and Saturday, March 23 and 24.

Those of us responsible for arrangements for the meeting are pushing ahead to complete the details. We can make the arrangements but the success of the meeting will depend upon the strength of attendance and the enthusiasm generated. Surely, the chief benefit derived from these sectional meetings is the quickening of enthusiasm and the inspiration gained. I urge each department head in high school and college to strain a point and, if possible, to bring his staff, one and all.

Due to the suggestion of Dr. W. D. Cairns we are to have Dr. Arnold Dresden, President of the Mathematical Association of America, as our guest speaker. This is an especial opportunity to have the president of the Association with us. Let us give him a hearty welcome into the splendid fellowship of our sections of the council and of the association.

At the banquet on Friday night Dr. Dresden will speak on *"The Mathematical Association of America and American Mathematics."* On Saturday morning the following papers will be read:

"Trends of Higher Education," by Dr. C. D. Smith, Mississippi State College.

"Combinations of Abstract Spaces," by Dr. Dorothy McCoy, Belhaven College.

"On the Degree of the Highest Common Factor," by Dr. W. V. Parker, Mississippi Woman's College.

"Some Aspects of the Calculus of Variations," by Dr. Arnold Dresden, Swarthmore College.

Following Dr. Dresden's paper anyone so desiring will be invited to give any special short reports on research.

P. K. SMITH, *Chairman,*
La.-Miss. Section of the M. A. of A.

ENGLISH MATHEMATICIANS—By Miss Loreen Dyson, Bolton High School, Alexandria, La.

A MATHEMATICAL THEORY FOR THE FEATHER PATTERN IN THE GUINEA FOWL—By Dr. W. L. Duren, Tulane University, New Orleans, La.

HISTORY OF MATHEMATICAL ORGANIZATIONS OF LOUISIANA-MISSISSIPPI TERRITORY—By Miss Alene Thurman, Ruston High School, Ruston, La.

THE COLLEGE INSTRUCTOR OF MATHEMATICS AND THE HIGH SCHOOL GRADUATE—By Miss Ruth Johnson, Louisiana State University, Baton Rouge, La.

Improving the Teaching of College Mathematics

By May M. Beenken
State Teachers College, Oshkosh, Wisconsin

In this discussion we shall confine our attention to the four year college, as distinct from the graduate school of the university, and consider its purpose. The college must train the student to acquire knowledge that has contributed to the advance of civilization and to use it to advantage. It must train him in the processes involved in acquiring and using this knowledge. Teaching, then, is the chief function of the college. Other functions are by-products.

Mathematics, a subject universal in time, place and circumstance, should hold an important place in college education. Every major intellectual concern of man is a concern of mathematics. Roger Bacon spoke truly when he said, "Mathematics is the gate and key of the sciences. Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of this world. And what is worse, men who are thus ignorant are unable to perceive their own ignorance and do not seek a remedy." Every science, as it becomes more exact, uses more mathematics. The physical sciences have long depended upon mathematics, while the biological sciences are rapidly taking over mathematical methods. Even the social scientist admits that literary expression, alone, is inadequate to bring into the focus of his attention the many involved facts and relationships concerning human affairs. He too must look to mathematics for a workable symbolism.

Mathematics can claim a prominent place in teaching the student how to acquire and use knowledge. It is a subject which demands rigorous thinking and has been called the ideal and norm of all careful thinking. The poet, Goethe, has said, "Mathematics opens the fountain of all thought."

College courses in mathematics should acquaint the student with the contributions of mathematics to civilization. They should give him a working knowledge of mathematics which he may apply to the science, develop in him habits of clear thinking, and give him an esthetic appreciation of mathematical reasoning.

College teaching is effective in so far as it contributes to the purpose of the college. How is the college student being trained in

acquiring mathematical knowledge and in using this chief instrument for penetrating into the secrets of nature? We need only look about us to find evidences of inefficiency. We find young instructors who know mathematics, but who are floundering about in a hopeless endeavor to teach freshmen. Some are busy at work doing research and begruding the time needed to teach undergraduate classes. There are older professors teaching mechanically, year after year, the same courses in the same way, with no reference to any modern developments and applications. There are colleges in which freshman mathematics is so dull and unintelligible that few students elect advanced mathematics. Often the number of failures in freshman mathematics reaches as high as 40%. Not infrequently we find students who have been ill advised in their choice of subjects. Advanced students in the sciences are often handicapped because they were not advised to study the necessary mathematics during their college course. The mathematics professors had failed to interest themselves in the needs of these students.

Dr. Seidlin*, professor of mathematics at Alfred College, New York, made an investigation of actual classroom procedure employed by teachers of freshman and sophomore mathematics in 20 eastern colleges and universities. He observed 150 classroom recitations which he found could be classified into seven categories. These seven methods are listed here, in the order of desirability from the least to the most desirable, with the number of recitations of each type.

1. A ground-covering, text-book repeating recitation, at which students participate technically and mechanically (45).
2. A blackboard recitation, at which the students recite and get a grade (61).
3. The lecture which goes far afield, generally instructive and entertaining. The students, however interested, remain distinctly passive (8).
4. A carefully planned question and answer development (4).
5. The lecture used as a method of developing a theory, or a problem, or an exercise. The students participate "actively" in the proceedings (14).

*Seidlin, Joseph. A critical study of the teaching of elementary college mathematics.

6. A combination of blackboard and oral recitation in which students show a lively interest; the instruction is casual and entertaining (8).

7. The recitation at which the time is fairly evenly divided between instructor and students. The students' questions appear to be spontaneous. The instructor is resourceful and enthusiastic (10).

You will notice that 106 out of 150 observed recitations were of the two types rated least desirable by Seidlin. These two methods offer the teacher the path of least resistance. At schools in which these first two methods of instruction dominated, only 3% to 8% of the freshmen elected sophomore mathematics. At schools in which the last two methods dominated, 20% of the freshmen continued with sophomore mathematics.

In his observations Seidlin found that the fundamental principles of the learning process were frequently violated. The teaching of a subject was often justified just because it was in the text-book or because it was to be included in an examination. Some of the teachers, he observed, made serious mathematical errors, due either to ignorance or inexcusable carelessness. Forty-six such errors were recorded. Some were made by teachers noted for the quantity and quality of their published research.

You may justly ask, "But what is good teaching and who is to be the judge?" Most professors will admit that they teach better at some times than at others. This is an admission that the quality of teaching can be estimated.

In 1925 a study† was made with Rhodes scholars. Each was asked to think of the best college teacher he had ever had. He was then asked to check any of a list of statements believed to be true of this best college teacher. The statement receiving the highest vote was: "He expected more initiative and allowed more independence to students." The second place was tied between these two statements: "This teacher was more careful in organizing his subject than other teachers." "He seemed to appreciate better the difficulties of students." The next highest vote went to the statement, "There was more of an inspiration for clean, honest living in his teaching."

†U. S. Brooks, Rhodes scholars' ideal professor, School & Society 21:375-7 March 28, 1925.

These statements show that even men, of such caliber as to receive Rhodes scholarships, appreciate a well organized course and a teacher with a sympathetic understanding of his students. What then can be said of the average student? Surely, this contradicts the oft heard statement that college sudents are mature enough to learn in spite of poor teaching.

I think you will agree with me that there is room for improvement in the teaching of college mathematics and the improvement of teaching is largely a matter of improving the teachers. Let us consider how this improvement can be brought about.

How does research affect teaching? The sole purpose of the graduate school is research. There, the students are mature and capable of teaching themselves. Only a research worker can direct research and inspire these students. But, for the college, excellence in teaching is of greatest importance. Research should be encouraged as a reinforcement of teaching. A person with real gift for research may use it to enliven his teaching. Theoretically, research, or the extending of knowledge, is an aid to teaching which is the communication of knowledge. But practically, there are many factors to be considered. If a man's interest is wholly in a narrow field of research to the detriment of his teaching, such research, instead of enhancing the value of his teaching, is a hindrance. Students need the enthusiasm and inspiration of a teacher with the power of creation. But the narrow specialist, so occupied with his own research that he has no time for his students, is certainly not developing in them a spirit of research. On the contrary, because of his poor teaching and neglect of his work, he is creating a distaste for it.

In order to keep mentally alert, the college teacher of mathematics should himself be working and learning constantly. We may well harken to the words of J. W. Young in his retiring presidential address to the Mathematical Association of America. He said, "The sin of the mathematician is not that he doesn't do research, the sin is idleness, when there is work to be done. If there be sinners in my audience, I would urge them to sin no more. If your interest is in research, do that; if you are of a philosophical temperament, cultivate the gardens of criticism, evaluation, and interpretation; if your interest is historical, do your plowing in the field of history; if you have the insight to see simplicity in apparent complexity, cultivate the field of advanced mathematics from the elementary point of view; if you have the gift of

popular exposition, develop your abilities in that direction; if you have executive and organizing ability, place that ability at the disposal of your organization. Whatever your abilities there is work for you to do,—for the greater glory of mathematics.' And may I add, "Whatever you do, do it for the greater glory of teaching which is the chief purpose for which you are employed."

The successful mathematics teacher must not only be a master of his subject, with a broad knowledge of present day applications of mathematics, but he must have a genuine interest in the students he is teaching. Intelligent teaching calls for organization of subject matter with reference to the needs of the learner. Successful teaching cannot go on without reference to the needs and interests of the student body. The mere presence of a topic in a text-book is no reason for teaching it. The teacher must be able to give the student a justification for the mathematics he is teaching. The college student does not need motivation to th extent of the elementary pupil, but, if motivation is there, it is an advantage. Analytic geometry, for example, takes on a new interest when the students learn the numerous uses of the parabola, ellipse and hyperbola that are rarely mentioned in text-books. The physics, chemistry, biology or social science student becomes intensely interested in mathematics when he sees some applications to his favorite science.

Mathematics professors must take it upon themselves to inform the science students, and especially social science students, of the mathematics they will need. The older professors of social science may fail to use mathematics because of their lack of knowledge in this field. Their students should not be handicapped in the same way. The mathematics teacher needs to acquaint himself with the work of related departments.

The effective teacher must have a sympathetic appreciation of his students. Too many of our college instructorships and professorships are filled with people who cannot teach and have no desire to teach. This is unfortunate, since only the love of teaching and of scholarship is a justifiable motive for entering the teaching profession. The teacher must have sympathy, understanding and patience. More faculty members ought to realize that their most important contribution in life is not primarily the text-books or few pieces of research they have written, but their influence on the lives of the students they have taught.

College teachers of mathematics must give more thought to the teaching process. I believe that great teachers are born, not made,

but that conscious effort directed toward the end of good teaching is of much value. As a group, we college teachers of mathematics are too self-satisfied. We feel that our subject needs no justification. We have a tendency to hold ourselves aloof from the public schools, elementary and high schools. We imply by our actions that their affairs are no concern of ours. Unless our mathematics teachers in both high school and college awaken to present needs, they are going to find themselves in a sad state of affairs. In some states the public school curriculum is made without any consultation with mathematicians. Mathematics is being taken out of the high school in an age when more mathematics is needed. The more poorly mathematics is taught in the high school the more this elementary work must be taken over by the college. We are connected with the high school in another way. A large portion of our students are preparing to teach mathematics in the high school. No course in education is as effective as imitation. If these students of today are to become good teachers of mathematics, they need to be taught by inspiring teachers with an enthusiasm for teaching mathematics.

To improve teaching, then, more attention must be directed to good teaching. It should be given well deserved recognition and made in part the basis of promotion. Since the teacher must constantly give forth, he must likewise take in. Teaching burden must not be so great as to hinder the teacher's growth as a scholar in his field.

What training should be given to the prospective college teacher of mathematics? Since from 60% to 80% of Ph. D.'s enter the field of teaching, they should receive some professional training for their life work. The young instructor need not learn by trial and error at the expense of his students. Good teaching is of importance from the very beginning, especially since poor teaching may become a habit.

I do not believe that the Ph. D. candidates should be sent to the education department for some courses in education. There is, unfortunately, a lack of understanding between departments of education and academic departments. Neither seems to know the problems of the other. What we need is a seminar conducted by real mathematicians, research men, who are good teachers as well. They ought to discuss such problems as what courses should be offered in the colleges to meet the needs of all classes of students, organization of courses and methods of teaching. Opportunity should be given students to visit good teachers of undergraduates. Even practice teaching would be desirable. The head of a university graduate department must know

the teaching qualities of his graduate students. Other qualities may be desirable but they cannot justify the appointment of a poor teacher. Research men with no desire to teach should certainly not be appointed to college positions.

In conclusion, I would say that college teaching of mathematics can be improved by directing the attention of those already in the field to good teaching and by giving the Ph. D. candidate definite training for his teaching profession.

The Gauss Archive and the Complete Edition of His Collected Works, 1860-1933

By G. Waldo Dunnington, *M. A.*
Kansas City, Mo.

On March 5, 1902, a Gauss Archive was established in the rooms used by the great mathematician in the west wing of the Gottingen Observatory. He possessed a library very rich in works on astronomy, mathematics, physics and other natural sciences, as well as a large collection of maps (especially of Hanover), and also many works in ancient and modern literature, philosophy, etc. In May, 1856, the Gauss heirs sold this library to the government of Hanover for the University of Gottingen. All the titles not already in the University library were placed there and the remainder was set up in the west wing of the observatory. Theresa Gauss, youngest child of Carl Friedrich, donated her share from the sale of the library, 6,000 marks, to cover the costs of bookcases and binding. The library was administered by Johann Benedict Listing (1808-1882), professor of physics. He was followed by Ernest Christian Julius Schering (1833-1897) who had been a student under Gauss, became professor of mathematics in Gottingen, and from 1860 to his death was the editor of Gauss' Collected Works.

Immediately after Schering's death began the task of accurately cataloging Gauss' library and combining it with the observatory library. In this way the accession list grew from 3,769 to 5,865—a gain of 2,096 volumes. If we count brochures and other miscellaneous items, the total mounts to 11,424; the combining of the two libraries and the preparation of a catalogue kept two men busy for a year and

a half, so that in 1899 the library was in an orderly condition for the first time in forty-four years.

The archive contains as much of the manuscript material of a scientific nature by Gauss as it has been possible to collect. From year to year various individuals have generously sent in copies or originals of letters by (or to) Gauss, and other interesting records. Occasionally some valuable item must be purchased. A large collection of material of an intimate or personal nature is to be found in the municipal archives at Brunswick, his birthplace. Schering had been in charge of research on terrestrial magnetism, in addition to his other duties, and used these same rooms in the west wing of the observatory both for this work and the editing of Gauss' Works. When Schering died Wiechert became director of the geophysics department and Dr. Martin Brendel took over the general editorship of the collected works. The rooms in the observatory were too crowded for both men and the Gauss manuscripts were moved to a room in the basement of the university administration building. This was extremely unfortunate, for the editor found great difficulty in his work because the manuscripts were thus separated from Gauss' personal library. In nearly all his own books he entered copious marginalia, notes on the fly-leaf, etc.

In the winter of 1901 the geophysics department moved into a new building on the Hainberg near Gottingen and the "Gauss rooms" were again available. Felix Klein then started a movement to set up the Gauss Archive and Dr. Schwarzschild, the new director of the observatory, agreed to the use of the rooms for this purpose.

The Royal Society of Sciences of Gottingen subsidized the publication and had charge of the manuscripts during the period of publication of the collected works, but the ownership rests with the University. Thus in April, 1902, the manuscripts and library were reunited to form the Gauss Archive. The Hanoverian government bought the manuscripts from the family "insofar as they were of scientific content." Several important items escaped in this manner, for some unknown reason; perhaps they were not carefully sorted. In 1898 Professor Paul Stackel found Gauss' scientific diary covering eighteen years, an exceedingly important document, held among the family papers by a grandson living in Hamlin. In the same way the inaugural lecture on astronomy came to light. Before his death in 1927 this same grandson placed practically all his material on Gauss in the permanent keeping of the archive, which is now administered

by the director of the observatory, Dr. Heinrich Kienle, the successor of Professor Dr. L. Ambronn, who died June 8, 1930.

The archive is fitted up in such a way that all the manuscript material is to be found in a room on the first floor, the room in which Gauss died on February 23, 1855, which fact is memoralized by a copper tablet placed on the wall by King George of Hanover. Also to be seen in a glass case is the small black satin cap often worn in pictures of Gauss, various instruments he used, and other relics including several busts and an oil painting of him. In the two adjoining smaller rooms we find his personal library.

It thrills one to see this collected evidence of the genius and creative activity of an outstanding mind over a period of seventy years. It is perhaps unequalled in modern times. Everything is there—from the scribblings of an eight-year-old boy, on through the full years of scientific creation, to the papers which lay on his desk at his death. It seems that from early youth on Gauss must have been conscious of what valuable treasures he would leave to posterity. He preserved nearly everything for us. At the age of nineteen he began his scientific diary and carried it through a period of some years.

Most of the manuscript material consists of a huge quantity of loose sheets on which Gauss carried out investigations of all kinds. Many of them are written carefully in a fine, neat hand with full notes; others are notes, equations, formulas, etc., hurriedly written and thrown together, being more difficult to decipher. These sheets are arranged in a series of files and cases according to subject: theory of numbers, analysis, geometry, geodesy, physics, astronomy, etc. They are not in chronological order, and indeed many unfortunately cannot be dated. An interesting article is a small octavo notebook, 1798-1805, which he called "Schedae"; the word scheda was used in the sense of a rough-draught such as one enters in a day-book. The first notes in this book refer to the lemniscate function; others contain his early investigations on elliptic functions. Of an earlier date than the Schedae is Leiste's textbook on arithmetic and algebra; Gauss annotated his copy so copiously and quoted it so frequently in the scientific diary that it has been placed among the manuscripts.

Next come the manuals or larger, bound notebooks in which Gauss liked to enter his mature and more fully developed investigations, but always without any systematic arrangement. Notes on the most diverse subjects follow each other; the very careful handwriting is always admirable—in spite of a strong effort to save paper it is always

legible. These loose sheets fill about sixty cases; then come the proceedings of the survey of the Kingdom of Hanover and several ledgers with records of geodetic, physical, and astronomical observations. The voluminous correspondence fills about twenty cases with letters to Gauss and about ten cases with letters by him. Also in the archive are the manuscripts which he published during his lifetime, various scientific memoirs, etc. Several notebooks worked out by his students from his lecture courses have been preserved and show that Gauss nearly always held his lectures in an elementary form, i. e., they were fairly easy for his hearers to understand. The same cannot be said of one who tries to read his writings. He never lectured on a subject with which he was occupied at the time and he complained that he had to be thinking about something which did not interest him.

Schering edited the first six volumes of the Collected Works of Gauss, and made them models of perfection; he had the advantage of personal association as a young man and was very versatile. His widow placed his papers at the disposal of his successors. Dr. Brendel took over the general editorship and also the manuscripts dealing with astronomy; he issued the final edition of the Theoria motus (1906) and the perturbation calculations, etc. Practically all the correspondence of Gauss with other scientific men and also much of a personal character has been published in volumes separate from the collected works.

Professor Robert Fricke of Brunswick edited the three volumes on analysis and theory of numbers, and Professor Paul Stackel in Kiel the volume on geometry. Professors Borsch and Kruger of the Central Geodetic Institute in Berlin collaborated on the geodetic material, while Dr. Wiechert prepared the papers on mathematical physics. Felix Klein acted for the Royal Society of Gottingen as a director of the entire undertaking. Then the World War delayed the completion of the editor's efforts; however, volume ten (part 1) appeared in 1917 and part 2 in the post-war years. Schering's last volume appeared in 1874 and there was a long gap until the next one was published in 1900; nothing appeared between 1906 and 1917. This gives an idea of how slow and how thoroughly the task was executed. Professor Brendel returned home in 1917 after being held prisoner two years by the French; during his absence and since then Dr. Ludwig Schlesinger of Giessen has served as an editor. Felix Klein died in 1925 but the work went on. Professor Clemens Schaefer of Breslau edited part one of volume eleven which came out in 1927, and curiously enough part

two of volume eleven had already appeared in 1924, mostly the work of Dr. A. Galle in Potsdam, but it was enlarged in 1929. Volume twelve containing various fragments which had not found a place elsewhere, including Gauss' Atlas of Terrestrial Magnetism, also appeared in 1929. In 1933 supplementary material relating to Gauss' work in function theory was issued by Schlesinger and similar material on mechanics by Professor Geppert. With this, the monumental complete edition was finally brought to a close, after seventy-three years of labor by many men. During the years 1911-1920 Klein, Brendel, and Schlesinger collected and published *"Materials for a Scientific Biography of Gauss"*; these consisted of eight monographs in the form of essays on various phases of his scientific work by as many different authors.

It is a most singular fact that no full length biography of Gauss (both scientific and personal) as yet exists. The writer of these lines has been personally acquainted since 1915 with various of the Gauss descendants in America and beginning in 1925, when the above fact was impressed on me, I started my own collection of material, which has grown to large proportions, partly through the generous assistance of the descendants. In 1927, the sesquicentennial year of Gauss, I spent the summer in Brunswick and Gottingen, examining all the known sources and increasing my collection considerably. Since then this has continued. At present I have written up the first half of his life. The biography will contain numerous illustrations and other hitherto unpublished items. The project moves along almost as slow as the editing of his works, such a mass of documents must be consulted if the biography is to be complete, authentic, and critical. No assurance can be given as to exactly when such an undertaking will be completed.

In the meantime Professors Brendel and Schlesinger have decided to issue a full biography in Germany; the present writer is proud to have been chosen to collaborate with them. We shall cooperate fully with each other but my biography in English will be quite independent of theirs, which may be issued as a memorial volume in connection with the celebration of the Bicentennial of the founding of the University of Gottingen, in 1937, if not before.

On the Teaching of the Trigonometric Functions of Half and Double Angles

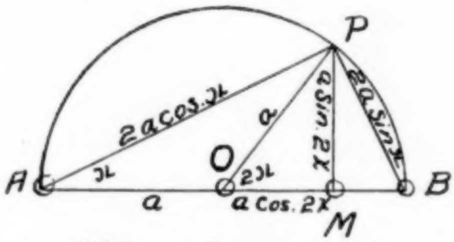
By Roscoe Woods
State University of Iowa

To create and maintain the interest of his pupils is the first problem with which a successful teacher concerns himself. While it is difficult to estimate the importance to be attached to this problem, it is common knowledge that it has no general solution. As a matter of fact, the manner in which interest is created and maintained varies with the individual teacher to such an extent that a method which is very successful in the hands of one teacher may prove of little worth in the hands of another. In spite of this fact, I am taking the liberty to offer, in what follows, some suggestions relative to the teaching of the trigonometric functions of the double and half angles. If these suggestions aid some teacher in formulating a successful plan for presenting this topic to his classes, the purpose of this note will have been fulfilled.

The average trigonometry in classroom use today devotes very little exposition to the topic under consideration. Fortunately this offers the teacher an opportunity to introduce it in his own way. This introduction should be of such a nature as to create interest. Since the average student is usually eager to learn that which is practical, the teacher may use this avenue to introduce this subject. I refer to the fact that we may use the formulas connecting the trigonometric functions of half and double angles to calculate the values of a series of trigonometric functions from the value of a single function. It usually appeals to the student to know that if we are given the value of $\sin x$, we can calculate $\sin(x/2)$, $\sin(x/4)$, etc., or $\sin 2x$, $\sin 4x$, and so on. Further mention may be made of the usefulness of these formulas in the study of the calculus, in the study of certain curves in higher mathematics, in the logarithmic solution of oblique triangles, and in analyzing wave motion in physics.

The usual method of deriving the formulas for $\sin 2x$, $\sin(x/2)$, and so on, is to set $y=x$ in the addition formulas $\sin(x+y)$, $\cos(x+y)$, and $\tan(x+y)$. While this is very simple for those who have had some experience in the study of trigonometry, it appears as so much magic to many beginners. The beginner is usually unable to see that far

ahead. At this point, it seems desirable to study some simple geometric figure in which there are two angles in the ratio 1:2. The following figure is convenient for this purpose and offers not only a review but the actual derivation of some of the formulas.:



$$AM = a(1 + \cos 2x).$$

$$MB = a(1 - \cos 2x).$$

From triangle AMP, we have,

$$\sin x = \frac{a \sin 2x}{2a \cos x}, \text{ or, } 2 \sin x \cos x = \sin 2x,$$

$$\cos x = \frac{a(1 + \cos 2x)}{2a \cos x}, \text{ or, } 2 \cos^2 x = 1 + \cos 2x,$$

$$\tan x = \frac{a \sin 2x}{a(1 + \cos 2x)}, \text{ or, } \tan x = \frac{\sin 2x}{1 + \cos 2x}.$$

From triangle PMB, we have,

$$\sin x = \frac{a(1 - \cos 2x)}{2a \sin x}, \text{ or, } 2 \sin^2 x = 1 - \cos 2x,$$

$$\cos x = \frac{a \sin 2x}{2a \cos x}, \text{ or, } 2 \sin x \cos x = \sin 2x,$$

$$\tan x = \frac{a(1 - \cos 2x)}{a \sin 2x}, \text{ or ,} \tan x = \frac{1 - \cos 2x}{\sin 2x}.$$

After the student has studied the above figure, the formulas for half and double angles may be derived analytically from the addition and subtraction formulas. The latter is used purposely, for by setting $y=x/2$ in the subtraction forumlas, $\sin(x-y)$ and $\cos(x-y)$, we may derive with ease two formulas which require a considerable amount of juggling otherwise.

After we have derived the formulas for the double angles, a set of drill exercises should be given. These drill exercises should contain angles of various types, such as 37°, 3x, 2x/3, 5x/7, and etc. The same procedure should be followed after the derivation of the half angle formulas.

In order to avoid the common mistake which many students often make of writing $2\sin x$ for $\sin 2x$, and $\sin x$ for $2\sin(x/2)$, several drill exercises should be given, in which not only such pairs of angles as 30° and 60°, 45° and 90°, are used, but pairs of angles for which a table of natural functions is necessary. In some way the table carries great weight with the beginning student, consequently, exercises of the following type may be added: Given $\sin(x/2-15°)=.785$, find x. This type of problem seems to ground the student in his thinking about half and double angles.

I am sure that many teachers will admit that some memory work is necessary. The kind and amount of memory work required is determined by the student. The good student needs to memorize the formulas only and not many of them. But there are many students who are able to quote a formula correctly, and who are not able to apply it when the symbol which has been memorized is changed to another. This fact suggests that such type of student should be trained not to memorize the formula but to memorize the formula translated into English. For example, the formula $\sin 2x=2\sin x\cos x$, translated into English reads: *The sine of two times an angle is equal to two times the sine of the angle times the cosine of the angle.*

There is another type of exercise which is helpful to furnish the student a method for solving numerical problems in half and double angles. This exercise is to teach the student to form the practice of expressing any given function of an angle not only in terms of the functions of an angle twice as large but also in terms of the functions of an angle half as large. For example, Given $\tan(3x/2)=7/24$, find $\sin 3x$. If the student now writes the two formulas: $\sin 3x=2\sin(3x/2)\cos(3x/2)$,

and $\sin 3x = \sqrt{\dfrac{1-\cos 6x}{2}}$, he has at his command the correct formula for solving the exercise. He has merely to reject theformula that does not apply.

Even though I use all the above illustrations, drills, and devices, including home written work, I find that many students do not master the simplest relations connecting half and double angles. There must be some underlying principle, or some inherent difficulty which I have not found. I am sure that I will not be successful until I have found this principle or difficulty and have devised a means of surmounting it.

Book Review Department

Edited by
P. K. SMITH

Trigonometry, by Crathorne and Lytle, copyright 1930 by Henry Holt and Company, Inc.

After a rather hasty review of this text the reviewer is impressed with the elegant appearance and the accuracy of the mathematical statements which are characteristic of earlier publications in which Professor Crathorne has had a part, such for example as the algebra series by Rietz and Crathorne. The usual topics are included, giving solutions of both the plane and spherical triangle. A chapter on complex numbers and hyperbolic functions appears, which in a way stands in contrast with the point of view of a well known calculus text where the hyperbolic function is purposely omitted.

As stated in the preface, two methods of approach to the theory of angles are recognized and the approach through the acute angle and its use is deliberately selected. With this view as a teaching technique the reviewer is in hearty accord. The point apparently is that a beginner who can be shown first the flag in a short hole may be inspired to work seriously at learning the game. We also find in the preface a detailed outline of lessons for a two semester hour course with suggestions for a longer course when required. The text is bound together with adequate tables, which, however, are

not bound separately. Although the text may be had without tables, this situation would probably inconvenience teachers who desire the use of tables apart from the text for use in black board drills and in written tests.

Problem lists are adequate and practical. The right triangle is treated directly, logarithms being reserved for the longer formulas where their use serves an obvious purpose. In chapter IV the question of computation and approximate tabulations is raised by very timely examples on selection of significant figures. This discussion might well have preceded the short table on page 13, since the treatment obviously applies to the use of that table. Chapter II carries approximately eighteen pages on the idea of extending functions of positive acute angles to other angles. This probably could be done better by a shorter and more unified treatment. We should also like to see problems and situations that illustrate practical use of the results.

At first sight chapter V, on solution of triangles, would appear short, but actual count reveals more than 100 exercises and problems in addition to adequate illustrations. The arrangement of material and the treatment as a whole bears evidence of consideration for the fundamental learning processes, and for this reason it should serve particularly well for teaching beginners in trigonometry. It is our opinion that the text ranks high among the large number of texts on this subject now available.

C. D. Smith.

Problem Department

Edited by
T. A. Bickerstaff

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

PROBLEMS FOR SOLUTION

No 51 Proposed by Henry Schroeder, Louisiana Polytechnic Institute.

Determine a point whose distances from three fixed points should have the minimum sum.

No. 52. Proposed by H. T. R. Aude, Colgate University.

Given a circle with the center at O and a point P not on the circle. A line is drawn through P cutting the circle in two points at which the tangents are drawn. These meet the line through P which is perpendicular to PO in the points C and D. Are PC and PD equal in length?

No. 53. Proposed by T. A. Bickerstaff.

Sum the series,

$$\operatorname{Sin} \alpha + \frac{1}{2} \sin 2\alpha + \frac{1}{2^2} \sin 3\alpha + \ldots\ldots\ \text{to infinity.}$$

No. 54. Proposed by H. T. R. Aude, Colgate University.

If a horse is tied by a rope 60 feet long, staked at the side of a vertical cylindrical tank of 100 feet diameter, find the area over which the horse can graze.

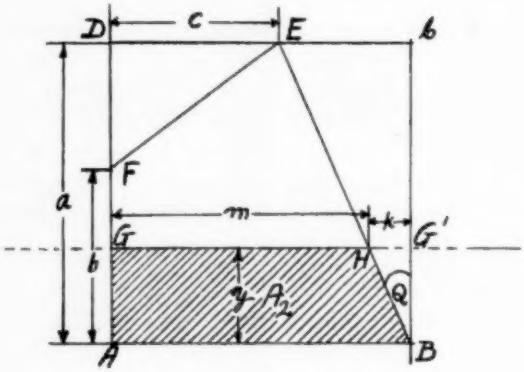
No. 46. Proposed by Professor C. D. Smith, Mississippi State College.

Given a square ABCD of side a and quadrilateral ABEF such that A and B are vertices of the square, E is on CD and F is on AD. Required, a formula for bisecting the quadrilateral by a line G G′ parallel to AB, and discuss the special cases where

$$F A \begin{matrix} < \\ = \\ > \end{matrix} G A$$

Solved by A. W. Randall, Prairie View State College, Prairie View, Texas.

Solution for FA > GA.



Let FA $= b$

DE $= c$

HG′ $= k$

GA $= y$, the

number of units above AB the line GG′ must be drawn to bisect quadrilateral ABEF. Take

$A_1 =$ area of quadrilateral ABEF.

$A_2 =$ area of one-half of ABEF.

From the figure,

$$A_1 = (a^2 + bc)/2.$$

Now the line GG′ is to divide the quadrilateral into two equal parts. Hence

$$A_2 = (a^2 + bc)/4.$$

The area of the quadrilateral AGHB is one-half quadrilateral ABEF, and is equal to $y(m+a)/2$.

Therefore

$$(m+a)y = (a^2 + bc)/2. \quad \ldots\ldots\ldots \quad (1)$$

$$m = a - k. \quad \ldots\ldots\ldots \quad (2)$$

$$k = y \tan Q. \quad \ldots\ldots\ldots \quad (3)$$

Substituting (3) in (2), we have

$$m = a - y \tan Q. \qquad (4)$$

Substituting (4) in (1), and reducing, we get

$$2y^2 \tan Q - 4ay + a^2 + bc = 0. \qquad (5)$$

Solving (5) for y, we obtain,

$$y = (2a - \sqrt{4a^2 - 2 \tan Q(a^2 + bc)})/2 \tan Q. \qquad (6)$$

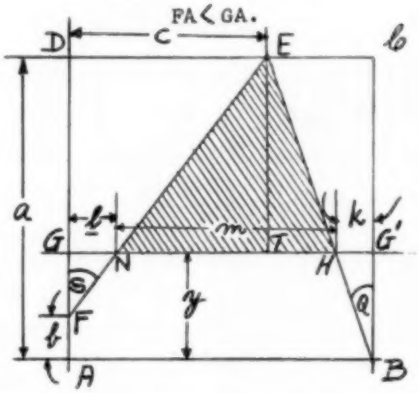
As a special case, let E coincide with C, and then

$$c = m = a. \qquad (7)$$

Substituting (7) in (1), we have

$$y = (a + b)/4. \qquad (8)$$

When



Take triangle NHE = one-half quadrilateral ABEF.

Then

$$m \,.\, ET = (a^2 + bc)/2. \qquad (1)$$

$$m = a - k - \underline{l}. \qquad (2)$$

$$k = y \tan Q. \qquad (3)$$

$$\underline{l} = (y - b) \tan S. \qquad (4)$$

Substituting (4) and (3) in (2), we have,

$$m = a - y \tan Q - y \tan S + b \tan S. \qquad ..(5)$$

$$ET = a - y. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (6)$$

Multiplying (5) and (6), we get

$$(a-y)(a-y \tan Q - y \tan S + b \tan S) = (a^2+bc)/2. \quad . \; . \; . \; . \qquad (7)$$

Clearing and reducing, we obtain

$$2y^2(\tan Q+\tan S) - 2y(a \tan Q + a \tan S + b \tan S + a) +$$
$$a^2 + 2ab \tan S - bc = O. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (8)$$

Solving (8) for y, we find

$$y = (2(a \tan Q + a \tan S + b \tan S + a(-\sqrt{4(a \tan Q + a \tan S + b \tan S + a)^2 - 8(\tan Q + \tan S)(a^2 + 2ab \tan S - bc)})/4(\tan Q + \tan S). \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (9)$$

If E coincides with C, we find as a special case,

$$y = (2(a \tan S + b \tan S + a) - \sqrt{4(a \tan S + \tan S + a)^2 - 8 \tan S (a^2 + 2ab \tan S - ab)})/4 \tan S. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (10)$$

When

$$FA = GA.$$

In this case we easily see that if we take angle

$$S = O$$

in (9), we get

$$y = (2(a \tan Q + a) - \sqrt{4(a \tan Q + a)^2 - 8 \tan Q (a^2 - bc)})/4 \tan Q. \quad (11)$$

Equation (11), under $FA = GA$, is identically the same equation as the one we would have obtained, had we derived y for the case $FA = GA$ from the figure.

No. 47. Proposed by E. C. Kennedy, University of Texas.

Prove or disprove,

$$\lim_{n \to \infty} \frac{\sqrt{0} + \sqrt{1} + \sqrt{2} + \dots + \sqrt{n}}{(n+1)\sqrt{n}} = \frac{2}{3}$$

Solved by J. D. Hill and M. F. Rosskopf, Brown University, Providence, R. I.

By constructing outer and inner rectangles on the unit intervals, it can be observed that if,

$$S_n = \sqrt{0} + \sqrt{1} + \sqrt{2} + \ldots\ldots + \sqrt{n}$$

Then, $$\int_0^n \sqrt{x}\,dx < S_n < \int_0^{n+1} \sqrt{x}\,dx$$

On evaluating these two integrals and dividing through by $(n+1)\sqrt{n}$ we obtain

$$\frac{2}{3}\,\frac{n}{n+1} < \frac{S_n}{(n+1)\sqrt{n}} < \frac{2}{3}\sqrt{\frac{n+1}{n}}$$

Since $$\frac{S_n}{(n+1)\sqrt{n}}$$

lies between two quantities each having the limit 2/3, it follows that,

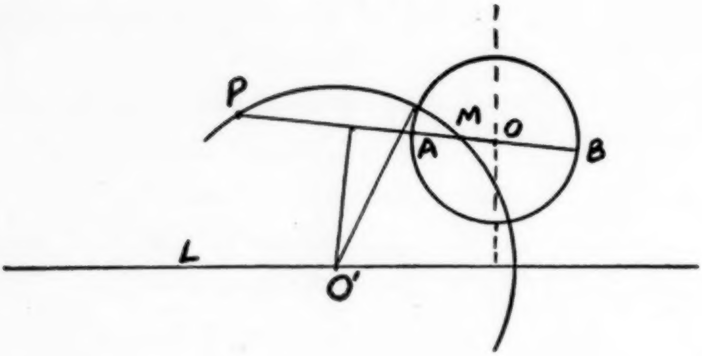
$$\lim_{n\to\infty} \frac{S_n}{(n+1)\sqrt{n}} = \frac{2}{3}$$

No. 49. Proposed by H. T. R. Aude, Colgate University.

Solution by Henry Schroeder, Louisiana Polytechnic Institute.

Also solved by A. W. Ransdall and by Richard A. Miller.

Find by construction the circle which fulfills the three conditions: Its center lies on a given line, it is orthogonal to a given circle, and it passes through a given point.



Construction: Given line L, circle O, and point P. Draw line from P through O, the center of circle, cutting circle at A and B. Locate M, the inverse of P with respect to A, B.

> **Definition**—"Two points collinear with the center of a circle and such that the product of their distances from the center is equal to the square of the radius are said to be inverse points with respect to the circle".

$$\overline{OA}^2 = OM \cdot OP$$

OA and OP are known; therefore we can determine OM.

Draw perpendicular bisector of PM. This line intersects the given line L at O′ the center of the required circle.

"If two circles are orthogonal, every diameter of one circle cuts the other circle in inverse points of the first." Theorem, page 146, Altshiller-Court College Geometry.

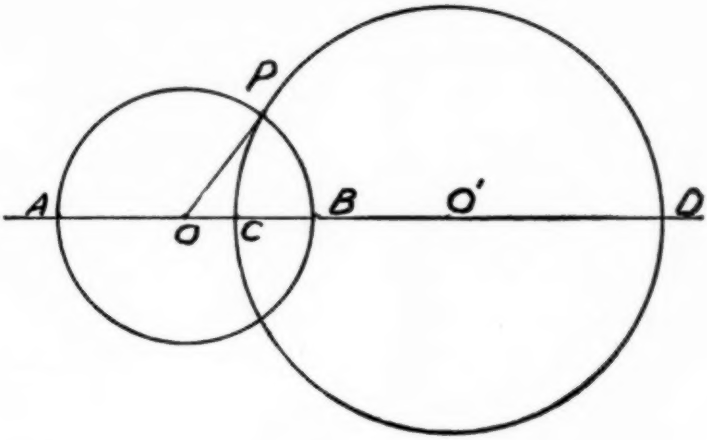
If P is placed on line perpendicular to L and through O, center of the given circle, it will be impossible to construct the required circle unless the perpendicular bisector of line joining P and its inverse point coincides with the given line L, in which case an infinite number of circles can be constructed.

No. 50. Proposed by H. T. R. Aude, Colgate University.

Two circles are orthogonal to each other. One circle cuts the line through the centers in the points A and B. The other circle

cuts it in the points C and D. Show that the points A and B separate the points C and D harmonically.

Solution by Richard A. Miller, University of Mississippi.



Given: Circles O and O′ orthogonal. ABD is line through centers of both circles, with O cutting ABD in points A and B, and O′ cutting ABD in points C and D.

To Prove: A and B separate C and D harmonically.

Proof: Draw OP as tangent to circle O′.

Circles O and O′ are orthogonal, hence radius OP is the tangent of O′.

Hence $OC \times OD = OP^2$ (Square of tangent to circle equals product of segments of secant from that point).

But $OP = OB$ Radii of same circle O.

Hence $OC \times OD = OB^2$

Therefore C, D are inverse points of A,B. (Two points collinear with center of a given circle such that product of their distances from center is equal to square of radius are "inverse points" with respect to circle.)

Hence (ABCD) is harmonic: (Two inverse points divide harmonically the diameter on which they lie.)

This problem is special case of theorem: "If two circles are orthogonal, every diameter of one circle cuts the other circle in inverse points of the first." Altschiller-Court, "College Geometry." p. 146.

Also solved by A. W. Randall and Henry Schroeder.

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